Conic Sections

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. Find the equation of the circle with centre (–3, 2) and radius 4.
(a) $(x + 3)^2 + (y - 2)^2 = 16$
(b) $(x - 3)^2 + (y - 2)^2 = 9$
(c) $(x + 3)^2 + (y + 2)^2 = 25$
(d) $x^2 + y^2 = 25$

2. Find the centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$.
(a) (5, 4), – 7 (b) (–4, –5), 7
(c) (4, –5), 7 (d) (5, –4), 7

3. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is
(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 - 2x - 2y - 1 = 0$
(c) $x^2 + y^2 - 2x - 2y = 0$
(d) $x^2 + y^2 - 2x + 2y - 1 = 0$

4. If the circle $x^2 + y^2 - 17x + 2fy + c = 0$ passes through (3, 1), (14, –1) and (11, 5), then c is
(a) 0 (b) –41 (c) –17/2 (d) 41

5. The equation of a circle with centre at (1, 0) and circumference 10 π units is
(a) $x^2 + y^2 - 2x + 24 = 0$
(b) $x^2 + y^2 - x - 25 = 0$
(c) $x^2 + y^2 - 2x - 24 = 0$
(d) $x^2 + y^2 + 2x + 24 = 0$

6. The equation of family of circles with centre at (h, k) touching the x-axis is given by
(a) $x^2 + y^2 - 2hx + h^2 = 0$
(b) $x^2 + y^2 - 2hx - 2ky + h^2 = 0$
(c) $x^2 + y^2 - 2hx - 2ky - h^2 = 0$
(d) $x^2 + y^2 - 2hx - 2ky = 0$

7. If (4, 0) is a point on the circle $x^2 + ax + y^2 = 0$, then the centre of the circle is at
(a) (–2, 0) (b) (0, 2) (c) (2, 0) (d) (1, 0)

8. Find the equation of the parabola with focus (2, 0) and directrix $x = -2$.
(a) $x^2 = 8y$ (b) $y^2 = 4x$ (c) $x^2 = 4y$ (d) $y^2 = 8x$

9. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).
(a) $x^2 = 8y$ (b) $x^2 = 2y$
(c) $y^2 = 4x$ (d) $y^2 = 8x$

10. Find the equation of the parabola which is symmetric about the y-axis, and passes through the point (2, –3).
(a) $x^2 = 4y$ (b) $4y = 3x^2$
(c) $3x^2 = -4y$ (d) $3y = -4x^2$

11. The equation of the parabola whose focus is the point (2, 3) and directrix is the line $x - 4y + 3 = 0$ is
(a) $12x^2 - y^2 + 4xy - 78y = 0$
(b) $14x^2 + y^2 + 8xy - 4x + 2y - 4 = 0$
(c) $12x^2 + y^2 + 4xy - 78x = 0$
(d) $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

12. The equation of the directrix of $(x - 1)^2 = 2(y - 2)$ is
(a) $2y + 3 = 0$ (b) $2x + 1 = 0$
(c) $2x - 1 = 0$ (d) $2y - 3 = 0$

13. If the vertex of the parabola $y = x^2 - 16x + K$ lies on x-axis, then the value of K is
(a) 16 (b) 8 (c) 64 (d) – 64

14. The distance between the vertex of the parabola $y = x^2 - 4x + 3$ and the centre of the circle $x^2 = 9 - (y - 3)^2$ is
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2}$ (d) $2\sqrt{5}$

15. If the equation of the parabola is $x^2 = -8y$, the equation of the directix and length of latus rectum, respectively are
(a) $y = 3, 4$ (b) $y = -2, 4$
(c) $y = 2, 8$ (d) $y = -2, 8$

16. The two ends of latus rectum of a parabola are the points (3, 6) and (– 5, 6). The focus is
(a) (1, 6) (b) (– 1, 6) (c) (1, – 6) (d) (–1, –6)

17. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is
(a) $x = -1$ (b) $x = 1$
(c) $x = -3/2$ (d) $x = 3/2$

18. If the ellipse $\frac{x^2}{4}+\frac{y^2}{1} = 1$ meets the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 - 10b + 25$, then the value of b does not satisfy

(a) $(-\infty, 4)$ (b) $[4, 6]$
(c) $(6, \infty)$ (d) None of these

19. The sum of the distances of any point on the ellipse $3x^2 + 4y^2 = 24$ from its foci is

(a) $4\sqrt{2}$ (b) 8 (c) $16\sqrt{2}$ (d) $2\sqrt{2}$

20. Find the coordinates of the foci and eccentricity respectively of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1.$

(a) $(0, \pm 4), \frac{4}{5}$ (b) $(\pm 4, 0), \frac{4}{5}$
(c) $(0, \pm 4), \frac{4}{3}$ (d) $(0, \pm 2), \frac{4}{5}$

21. If the length of the major axis of an ellipse is 17/8 times the length of the minor axis, then the eccentricity of the ellipse is

(a) $\frac{8}{17}$ (b) $\frac{15}{17}$ (c) $\frac{9}{17}$ (d) $\frac{2\sqrt{2}}{17}$

22. Find the equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directrices.

(a) $\frac{x^2}{36}+\frac{y^2}{11}=4$ (b) $\frac{x^2}{6}+\frac{y^2}{11}=1$
(c) $\frac{x^2}{36}+\frac{y^2}{11}=1$ (d) $\frac{x^2}{36}+\frac{y^2}{4}=1$

23. The equation of the ellipse whose centre is at the origin and the x-axis, the major axis, which passes through the points $(-6, 1)$ and $(4, -4)$ is

(a) $3x^2 - 4y^2 = 32$ (b) $3x^2 + 4y^2 = 112$
(c) $4x^2 - 3y^2 = 112$ (d) $4x^2 + 3y^2 = 112$

24. If the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ are $(0, \sqrt{7})$ and $(0, -\sqrt{7})$, then the foci of the ellipse $\frac{x^2}{9+t^2}+\frac{y^2}{16+t^2}=1,\ t \in R$, are

(a) $(0, \sqrt{7}), (0, -\sqrt{7})$ (b) $(0, 7), (0, -7)$
(c) $(0, 2\sqrt{7}), (0, -2\sqrt{7})$ (d) $(\sqrt{7}, 0), (-\sqrt{7}, 0)$

25. The equation of the ellipse whose focus is $(1, -1)$, directrix is the line $x - y - 3 = 0$ and the eccentricity is $\frac{1}{\sqrt{2}}$, is

(a) $3x^2 + 2xy + 3y^2 - 2x + 2y - 1 = 0$
(b) $3x^2 + 2xy + 3y^2 + 2 = 0$
(c) $3x^2 + 2xy + 3y^2 + 2x - 2y - 1 = 0$
(d) None of these

26. The eccentricity of the ellipse $12x^2 + 7y^2 = 84$ is equal to

(a) $\frac{\sqrt{5}}{7}$ (b) $\sqrt{\frac{5}{12}}$ (c) $\frac{\sqrt{5}}{12}$ (d) $\frac{5}{7}$

27. If for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, y-axis is the minor axis and the length of the latus rectum is one half of the length of its minor axis, then its eccentricity is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{4}$

28. The eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose length of latus rectum is half of the length of its major axis is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{\frac{2}{3}}$
(c) $\frac{\sqrt{3}}{2}$ (d) None of these

29. The length of the latus rectum of the ellipse $9x^2 + 16y^2 = 144$ is

(a) 4 (b) 11/4 (c) 7/2 (d) 9/2

30. The sum of the distances of a point $(2, -3)$ from the foci of an ellipse $16(x - 2)^2 + 25(y + 3)^2 = 400$ is

(a) 8 (b) 6 (c) 50 (d) 32

31. The eccentricity of the conic

$\frac{(x+2)^2}{7}+(y-1)^2=1$ is

(a) $\sqrt{\frac{7}{8}}$ (b) $\sqrt{\frac{6}{17}}$ (c) $\sqrt{\frac{6}{7}}$ (d) $\sqrt{\frac{6}{11}}$

32. The equation $\frac{x^2}{14-a}+\frac{y^2}{9-a}=1$ represents a/an

(a) ellipse if $a > 9$ (b) hyperbola if $9 < a < 14$
(c) hyperbola if $a > 14$ (d) ellipse if $9 < a < 14$

33. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, \quad |r| < 1$ represents a/an

(a) ellipse (b) hyperbola
(c) circle (d) None of these

34. The eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$ is

(a) $\frac{1}{\sqrt{13}}$ (b) $\sqrt{13}$ (c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{13}}{3}$

35. If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic $9x^2 - 4y^2 = 36$ then which is true ?

(a) $e_1^2 + e_2^2 = 2$ (b) $3 < e_1^2 + e_2^2 < 4$
(c) $e_1^2 + e_2^2 > 4$ (d) None of these

36. The eccentricity of the hyperbola $x^2 - y^2 = 2004$ is

(a) $\sqrt{3}$ (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{2}$

37. The equation of hyperbola referred to its axes as axes of co-ordinate whose distance between the foci is 20 and eccentricity equals $\sqrt{2}$ is

(a) $x^2 - y^2 = 25$ (b) $x^2 - y^2 = 50$
(c) $x^2 - y^2 = 125$ (d) $x^2 + y^2 = 25$

38. The eccentricity of the hyperbola $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by

(a) $e = \sqrt{\frac{a^2+b^2}{a^2}}$ (b) $e = \sqrt{\frac{a^2-b^2}{a^2}}$

(c) $e = \sqrt{\frac{b^2-a^2}{a^2}}$ (d) $e = \sqrt{\frac{a^2+b^2}{b^2}}$

39. Find the coordinates of the foci and the length of the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

(a) $(0, \pm 2), \frac{32}{3}$ (b) $(0, \pm 5), \frac{32}{3}$

(c) $(\pm 5, 0), \frac{32}{3}$ (d) $(0, \pm 5), \frac{3}{32}$

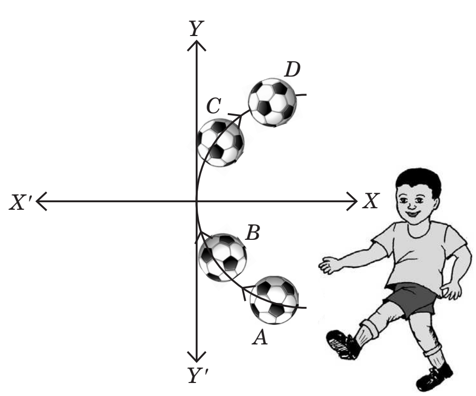
40. The equation of the hyperbola with vertices (3, 0), (– 3, 0) and semi-latus rectum 4 is given by

(a) $4x^2 - 3y^2 + 36 = 0$ (b) $4x^2 - 3y^2 + 12 = 0$
(c) $4x^2 - 3y^2 - 36 = 0$ (d) None of these

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Arun was playing a football match. When he kicked the football, the path formed by the football from ground level is parabolic, which is shown in the following graph. Consider the coordinates of point A as (3, –2).



41. The equation of path formed by the football is

(a) $y^2 = x + 1$ (b) $3x^2 = 4y$
(c) $3y^2 = 4x$ (d) $x^2 = y - 1$

42. The equation of directrix of path formed by football is

(a) $x - \frac{4}{3} = 0$ (b) $x + \frac{2}{3} = 0$

(c) $x + 3 = 0$ (d) $x + \frac{1}{3} = 0$

43. The extremities of latus rectum of given curve are

(a) $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$ (b) $\left(\frac{2}{3}, \pm\frac{1}{3}\right)$

(c) $\left(\pm\frac{1}{3}, 1\right)$ (d) $\left(\pm\frac{1}{3}, \frac{4}{3}\right)$

44. The length of latus rectum of given curve is

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) 3 (d) $\frac{4}{3}$

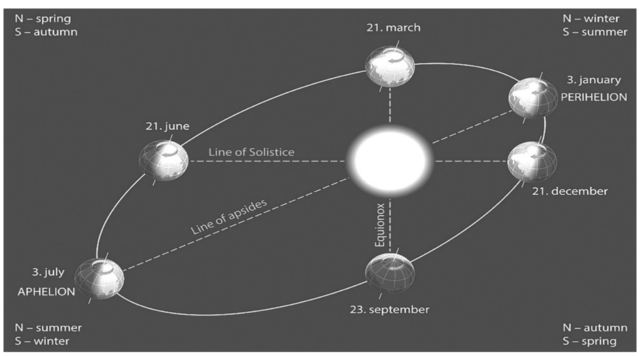
45. Which of the following point lies on the path formed by football?

(a) $(-2, 0)$ (b) $\left(1, \frac{3}{4}\right)$

(c) $\left(\frac{-1}{3}, 4\right)$ (d) $\left(\frac{3}{4}, 1\right)$

Case II : Read the following passage and answer the questions from 46 to 50.

In maths period, Ananya studied about the ellipse. On the same day, her Physics teacher was exploring the rotation of Earth around the Sun, where the Sun is situated at one of the foci of ellipse formed by Earth. Consider the path covered by Earth is represented by equation $\frac{x^2}{169} + \frac{y^2}{144} = 1$.



46. The vertices of the path covered by Earth around Sun is

(a) $(\pm 5, 0)$ (b) $(\pm 13, 0)$

(c) $(0, \pm 13)$ (d) $(0, \pm 12)$

47. The sum of focal radii of given curve is

(a) 36 (b) 24 (c) 26 (d) 16

48. The eccentricity of the given curve is

(a) $\frac{13}{5}$ (b) $\frac{12}{13}$ (c) $\frac{4}{13}$ (d) $\frac{5}{13}$

49. Equation of directrices of given curve are

(a) $y = \pm\frac{144}{169}$ (b) $y = \pm\frac{169}{5}$

(c) $x = \pm\frac{5}{169}$ (d) $x = \pm\frac{169}{5}$

50. Which of the following point lies on the curve formed by Earth around the Sun?

(a) $(-13, 14)$ (b) $(0, -12)$

(c) $(12, 0)$ (d) $(12, -13)$

Case III : Read the following passage and answer the questions from 51 to 55.

Karan, the student of class XI was studying in his house. He felt hungry and found that his mother was not at home. So, he went to the nearby shop and purchased a packet of chips. While eating the chips, he observed that one piece of the chips is in the shape of hyperbola. Consider the vertices of hyperbola at $(\pm 5, 0)$ and foci at $(\pm 7, 0)$.

51. The equation of hyperbolic curve formed by given piece of chips is

(a) $\frac{x^2}{15} - \frac{y^2}{16} = 1$ (b) $\frac{x^2}{25} - \frac{y^2}{24} = 1$

(c) $\frac{x^2}{24} - \frac{y^2}{16} = 1$ (d) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

52. The length of conjugate axis of given curve formed by given piece of chips is

(a) 25 (b) $8\sqrt{6}$ (c) 10 (d) $4\sqrt{6}$

53. The eccentricity of hyperbolic curve formed by given piece of chips is

(a) $\frac{6}{5}$ (b) $-\frac{7}{5}$

(c) $\frac{7}{5}$ (d) $-\frac{6}{5}$

54. What is the length of latus rectum of given hyperbolic curve?

(a) 9.6 (b) 4.8

(c) 10.9 (d) 5.4

55. The equation of directrices of given hyperbolic curve are

(a) $y = \pm\frac{7}{25}$ (b) $x = \pm\frac{24}{7}$

(c) $x = \pm\frac{25}{7}$ (d) $y = \pm\frac{12}{25}$

Assertion & Reasoning Based MCQs

Directions (Q.-56 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

56. Assertion : The sum of focal distances of a point on the ellipse $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ is 4.

Reason : The equation $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ can be expressed as $9(x - 1)^2 + 4(y - 3)^2 = 36$.

57. Assertion : The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10, respectively.

Reason : The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.

58. If the distances of foci and vertex of hyperbola from the centre are c and a respectively, then

Assertion : Eccentricity is always less than 1.

Reason : Foci are at a distance of ae from the centre.

59. Assertion : A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.

Reason : The point of intersection of parabola with the axis is called the vertex of the parabola.

60. Parabola is symmetric with respect to the axis of the parabola.

Assertion : If the equation of standard parabola has a term y^2, then the axis of symmetry is along the x-axis.

Reason : If the equation of standard parabola has a term x^2, then the axis of symmetry is along the x-axis.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Find the equation of the circle which passes through the point (4, 5) and has its centre at (2, 2).

2. Find the centre of the circle $x^2 + y^2 + 2y = 0$.

3. Find the equation of a parabola with vertex at origin and axis along x-axis and passing through the point (2, 8).

4. If the distance of the focus of a parabola from its directrix is 4, find the length of the latus rectum.

5. Find the coordinate of the focus of the parabola $y^2 = 12x$.

6. Find the eccentricity of the ellipse :

$$\frac{x^2}{25}+\frac{y^2}{9}=1.$$

7. Find the equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directices.

8. What is the eccentricity of the curve $4x^2 + y^2 = 100$?

9. What is the eccentricity of hyperbola whose vertices and foci are $(\pm 2, 0)$ and $(\pm 3, 0)$ respectively?

10. What is the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$?

Short Answer Type Questions (SA-I)

11. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

12. Find the equation of a circle whose centre is (1, –2) and which passes through the centre of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$.

13. Find the co-ordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum of the parabola $x^2 = -9y$.

14. Find the eccentricity of the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$.

15. The foci of an ellipse are (±2, 0) and its eccentricity is $\frac{1}{3}$. Find the equation of ellipse.

16. Write the equation of ellipse whose vertices are (±5, 0) and foci (±4, 0).

17. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find the latus rectum of the ellipse.

18. Find the equation of the hyperbola satisfying the given conditions: vertices (0, ± 5), foci (0, ± 8).

19. Find the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which passes through the points (3,0) and $(3\sqrt{2}, 2)$.

20. Find the equation of the hyperbola whose foci are $(\pm 3\sqrt{5}, 0)$ and the length of latus rectum is 8 units.

Short Answer Type Questions (SA-II)

21. If the abscissae and the ordinates of two points A and B be the roots of $ax^2 + bx + c = 0$ and $a'\,y^2 + b'\,y + c' = 0$ respectively, show that the equation of the circle described on AB as diameter is $aa'\,(x^2 + y^2) + a'bx + ab'\,y + (ca' + c'a) = 0$.

22. Find the equation of the circle which passes through the points (2, –2) and (3, 4) and whose centre lies on $x + y = 1$

23. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre lies on the line $4x + y = 16$.

24. Find the equation of the circle whose centre lies on the line $x - 4y = 1$ and which passes through the points (3, 7) and (5, 5).

25. Find the equation of the parabola whose focus is at (–1, –2) and the directrix is the line $x - 2y + 3 = 0$.

26. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

27. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

28. Find the equation of ellipse with centre at origin, major axis along x-axis, foci (±2, 0) and passing through the point (2, 3).

29. Find the equation of ellipse whose foci are (0, ±6) and length of minor axis is 16 units. Also find the coordinates of the points where the ellipse cuts y-axis and its latus rectum.

30. Find the equation of ellipse with centre at origin, major axis along the x-axis and passing through the points (4, 3) and (1, 4).

31. Find the equation of the ellipse with focus at (1, 1) and eccentricity $\frac{1}{2}$ and directrix $x - y + 3 = 0$. Also, find the equation of its major axis.

32. Find the equation of the ellipse whose focus is (1, 0), the directrix is $x + y + 1 = 0$ and eccentricity is equal to $1/\sqrt{2}$.

33. Find the equation of the hyperbola, the length of whose latus rectum is 8 and eccentricity is $\frac{3}{\sqrt{5}}$.

34. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the equation of the hyperbola if its eccentricity is 2.

35. Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

Long Answer Type Questions (LA)

36. Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$.

37. Find the vertex, focus, directrix, axis and latus-rectum of the parabola $y^2 = 4x + 4y$.

38. Find the equation of the parabola whose focus is (1, 1) and tangent at the vertex is $x + y = 1$.

39. If P is any point on a hyperbola and N is the foot of the perpendicular from P on the transverse axis, then prove that $\frac{(PN)^2}{(AN)(A'N)}=\frac{b^2}{a^2}$.

40. If e and e' be the eccentricities of a hyperbola and its conjugate, then prove that $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(a) :** Here $h = -3$, $k = 2$ and $r = 4$. Therefore, the equation of the required circle is

$$(x + 3)^2 + (y - 2)^2 = 16$$

2. **(b) :** The given equation is $(x^2 + 8x) + (y^2 + 10y) = 8$
Now, completing the squares within the parenthesis, we get

$$(x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$$
$$\Rightarrow (x + 4)^2 + (y + 5)^2 = 49 \Rightarrow (x - (-4))^2 + (y - (-5))^2 = 7^2$$

Therefore, the given circle has centre at (–4, –5) and radius 7.

3. **(a) :** Since the equation $(x - 1)^2 + (y - 1)^2 = 1$ represents a circle touching both the axes with its centre (1, 1) and radius one unit, therefore required equation is $x^2 + y^2 - 2x - 2y + 1 = 0$.

4. **(d) :** Given equation of circle is

$$x^2 + y^2 - 17x + 2fy + c = 0.$$

Since it passes through (3, 1), (14, –1) and (11, 5)

$\therefore$ $9 + 1 - 51 + 2f + c = 0$ or $2f + c = 41$ ...(i)

and $196 + 1 - 238 - 2f + c = 0$ or $-2f + c = 41$ ...(ii)

Adding (i) and (ii), we get

$$2c = (2 \times 41) \text{ or } c = 41.$$

5. **(c) :** Centre (1, 0), circumference = 10π (Given)

$\therefore$ $2\pi r = 10\pi \Rightarrow r = 5$

So, equation of circle is $(x - 1)^2 + (y - 0)^2 = 25$

$\Rightarrow$ $x^2 + y^2 - 2x - 24 = 0$.

6. **(b) :** When circle with centre (h, k) touches x-axis, the radius $(r) = k$

$\therefore$ Required equation of circle is

$$(x - h)^2 + (y - k)^2 = k^2$$
$$\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = k^2$$
$$\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 = 0$$

7. **(c) :** Since, (4, 0) is a point on the circle $x^2 + ax + y^2 = 0$.

$\therefore$ $16 + 4a + 0 = 0 \Rightarrow 4a = -16 \Rightarrow a = -4$

$\therefore$ Equation of circle is $x^2 - 4x + y^2 = 0$

or $x^2 - 4x + 4 - 4 + y^2 = 0 \Rightarrow (x - 2)^2 + (y - 0)^2 = 4$

$\therefore$ Centre (2, 0) and radius = 2

8. **(d) :** Since the focus (2, 0) lies on the positive x-axis and the directrix $x = -2$ is perpendicular to x-axis so the parabola is of the form $y^2 = 4ax$. Here, $a = 2$, therefore the required equation is $y^2 = 4(2)x = 8x$.

9. **(a) :** Since the vertex is at (0, 0) and the focus is at (0, 2) which lies on y-axis, the y-axis is the axis of the parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$. Here, we have $a = 2$, therefore required equation is $x^2 = 4(2)y$, *i.e.*, $x^2 = 8y$.

10. **(c) :** Since the parabola is symmetric about y-axis therefore its equation is of the form $x^2 = 4ay$ or $x^2 = -4ay$, where the sign depends on whether the parabola opens upwards or downwards. But the parabola passes through (2, –3) which lies in the fourth quadrant so it must open downwards. Thus the equation is of the form $x^2 = -4ay$.
Since the parabola passes through (2, –3), we have

$$2^2 = -4a(-3), \text{ i.e., } a = \frac{1}{3}$$

Therefore, the equation of the parabola is

$$x^2 = -4\left(\frac{1}{3}\right)y, \text{ i.e., } 3x^2 = -4y.$$

11. **(d) :** Using the definition of parabola, we have

$$\sqrt{(x-2)^2+(y-3)^2}=\left|\frac{x-4y+3}{\sqrt{17}}\right|$$

Squaring, we get

$17(x^2 + y^2 - 4x - 6y + 13) = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$

or $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

12. (d) : Equation of directrix of parabola $x^2 = 4ay$ is $y = -a$.

$\therefore \quad y - 2 = -\frac{1}{2}$ or $2(y - 2) = -1 \Rightarrow 2y - 3 = 0$

13. (c) : Given, $y = x^2 - 16x + K$

$\Rightarrow \quad y = (x - 8)^2 + (K - 64)$

Since vertex lies on x-axis, $y = 0$.

$\therefore \quad (x - 8)^2 + (K - 64) = 0$

It can be possible only when $x = 8$ and $K = 64$.

14. (d) : Here, $y = x^2 - 4x + 3$

$\Rightarrow \quad y + 1 = x^2 - 4x + 4 \Rightarrow y + 1 = (x - 2)^2$

So, the vertex is $(2, -1)$.

And for the circle, $x^2 + (y - 3)^2 = 9$, the centre is $(0, 3)$.

$\therefore$ Distance between vertex and centre

$= \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$

15. (c) : Comparing the given equation with standard form $x^2 = -4ay$, we get $a = 2$.

Therefore, the equation of directrix is $y = 2$ and the length of the latus rectum is $4a$, *i.e.*, 8.

16. (b) : Focus is the mid point of the latus rectum. So, its coordinates are $\left(\frac{3-5}{2}, \frac{6+6}{2}\right)$ or $(-1, 6)$.

17. (d) : We can write the given equation of the given parabola as $(y+2)^2 = -4\left(x - \frac{1}{2}\right)$

Shifting the origin at $(1/2, -2)$, the equation of parabola becomes $Y^2 = -4X$, where $X = x - 1/2$, $Y = y + 2$.

The equation of its directrix is $X = 1$.

Hence, required equation of directrix is $x - \frac{1}{2} = 1$

or $x = 3/2$.

18. (b) : Now according to condition $a > 1$

$\Rightarrow \quad b^2 - 10b + 25 = a > 1 \Rightarrow b^2 - 10b + 24 > 0$

$\Rightarrow \quad (b - 4)(b - 6) > 0 \Rightarrow b < 4$ or $b > 6 \Rightarrow b \notin [4, 6]$

19. (a) : Given equation of ellipse is $\frac{x^2}{8} + \frac{y^2}{6} = 1$

$\Rightarrow \quad a^2 = 8 \Rightarrow a = 2\sqrt{2}$

Sum of distances of any point on the ellipse from its foci $= 2a = 2 \times 2\sqrt{2} = 4\sqrt{2}$.

20. (b) : From the given equation of ellipse, we have $a = 5$ and $b = 3$.

$\therefore \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{4}{5}$

Therefore, the coordinates of the foci are $(\pm ae, 0)$ *i.e.*, $(\pm 4, 0)$

21. (b) : Length of major axis $= \frac{17}{8} \times$ length of minor axis

$\Rightarrow \quad 2a = \frac{17}{8} \times 2b \Rightarrow \frac{b}{a} = \frac{8}{17}$

$\Rightarrow \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$

22. (c) : We have $ae = 5$, $\frac{a}{e} = \frac{36}{5}$, which gives $a = 6$ and $e = \frac{5}{6}$.

Now $b = a\sqrt{1 - e^2} = 6\sqrt{1 - \frac{25}{36}} = \sqrt{11}$.

Thus, the equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{11} = 1$.

23. (b) : Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of the ellipse, $a > b$. Then according to the given conditions, we have

$\frac{36}{a^2} + \frac{1}{b^2} = 1 \quad \ldots(i)$ and $\frac{16}{a^2} + \frac{16}{b^2} = 1 \quad \ldots(ii)$

Solving (i) and (ii), we get

$a^2 = \frac{112}{3}$ and $b^2 = \frac{112}{4}$.

Hence, required equation of ellipse is $3x^2 + 4y^2 = 112$.

24. (a) : In the equation of ellipse $\frac{x^2}{9 + t^2} + \frac{y^2}{16 + t^2} = 1$, $b > a$. So, its foci will be $(0, \pm be)$.

Now $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9 + t^2}{16 + t^2}} = \frac{\sqrt{7}}{\sqrt{16 + t^2}}$

$\therefore \quad$ Foci $\equiv (0, \sqrt{7}), (0, -\sqrt{7})$

25. (a) : Let $P(x, y)$ be any point on the ellipse. Then by definition

$\sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{\sqrt{2}}\left|\frac{x - y - 3}{\sqrt{2}}\right|$

$\Rightarrow \quad (x - 1)^2 + (y + 1)^2 = \frac{1}{4}(x - y - 3)^2$

$\Rightarrow \quad 3x^2 + 2xy + 3y^2 - 2x + 2y - 1 = 0$

26. (b) : Given equation of ellipse is $12x^2 + 7y^2 = 84$

or $\frac{x^2}{\left(\frac{84}{12}\right)} + \frac{y^2}{\left(\frac{84}{7}\right)} = 1 \Rightarrow \frac{x^2}{(\sqrt{7})^2} + \frac{y^2}{(\sqrt{12})^2} = 1$

$\therefore \quad a = \sqrt{12}, b = \sqrt{7}$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{7}{12}} = \sqrt{\frac{5}{12}}$

27. (c) : Given that $\frac{2b^2}{a} = \frac{1}{2} \cdot 2b \Rightarrow \frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2}$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

28. (a) : It is given that $\frac{2b^2}{a} = \frac{1}{2} \times 2a$

$\Rightarrow \quad 2b^2 = a^2 \Rightarrow 2a^2(1 - e^2) = a^2 \Rightarrow e = 1/\sqrt{2}$.

29. (d) : Given equation of ellipse is $9x^2 + 16y^2 = 144$

or $\frac{9}{144}x^2 + \frac{16}{144}y^2 = 1$ or $\frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = 1$

$\therefore \quad a = 4, b = 3.$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 9}{4} = \frac{9}{2}$

30. (b) : Given that $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{16} = 1$

$\Rightarrow \quad a^2 = 25 \Rightarrow a = 5$ and $b^2 = 16 \Rightarrow b = 4$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Given point (2, – 3) is the centre of ellipse, so sum of distances of two foci from centre is

$2ae = 2 \times 5 \times \frac{3}{5} = 6$

31. (c) : Given, $\frac{(x+2)^2}{7} + (y-1)^2 = 1$

$\Rightarrow \quad \frac{(x+2)^2}{(\sqrt{7})^2} + \frac{(y-1)^2}{(1)^2} = 1$

It is an equation of ellipse $\therefore \quad a = \sqrt{7}, b = 1$

Eccentricity = $\sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{6}{7}}$

32. (b) : Given, $\frac{x^2}{14-a} + \frac{y^2}{9-a} = 1$

The equation will represent an ellipse if $14 - a > 0$ and $9 - a > 0$

$\Rightarrow \quad a < 14$ and $a < 9 \Rightarrow a < 9$

a hyperbola if $14 - a > 0$ and $9 - a < 0$

$\Rightarrow \quad a < 14$ and $a > 9 \Rightarrow 9 < a < 14$

33. (b) : Since $|r| < 1$, $1 - r$ and $1 + r$ are both positive. So we put $1 - r = a^2$, $1 + r = b^2$. Then the given equation becomes, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which represents a hyperbola.

34. (d) : Given that the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the points (3, 0) and $(3\sqrt{2}, 2)$, so we get $a^2 = 9$ and $b^2 = 4$.

Again, we know that $b^2 = a^2(e^2 - 1)$. This gives

$4 = 9(e^2 - 1) \Rightarrow e^2 = \frac{13}{9} \Rightarrow e = \frac{\sqrt{13}}{3}$

35. (b) : We have, $\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad \therefore \quad e_1^2 = \frac{9-4}{9} = \frac{5}{9}$

and $\frac{x^2}{4} - \frac{y^2}{9} = 1 \quad \therefore \quad e_2^2 = \frac{4+9}{4} = \frac{13}{4}$

Now $e_1^2 + e_2^2 = \left(\frac{5}{9}\right) + \left(\frac{13}{4}\right) = \frac{137}{36}$ > 3 but < 4

$\therefore \quad 3 < e_1^2 + e_2^2 < 4$

36. (d) : Given equation of hyperbola is $x^2 - y^2 = 2004$.

or $\frac{x^2}{(\sqrt{2004})^2} - \frac{y^2}{(\sqrt{2004})^2} = 1 \therefore a = \sqrt{2004}$ and $b = \sqrt{2004}$

Since $e^2 = 1 + \frac{b^2}{a^2}$, $e^2 = 1 + \frac{2004}{2004} = 1 + 1 = 2 \therefore e = \sqrt{2}$

37. (b) : Given, $2ae = 20$ and $e = \sqrt{2} \quad \therefore \quad a = \frac{20}{2e} = 5\sqrt{2}$

Again $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 25 \times 2(2 - 1) = 50$

$\therefore$ Equation of hyperbola is $x^2 - y^2 = 50$

38. (d) : $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \therefore a^2 = b^2(e^2 - 1) \Rightarrow e = \sqrt{\frac{a^2 + b^2}{b^2}}$.

39. (c) : We have, $a = 3, b = 4$ and $c = \sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5$

Therefore, the coordinates of the foci are (±5, 0).

Also, length of latus rectum = $\frac{2b^2}{a} = \frac{32}{3}$

40. (c) : We have $a = 3$ and $b^2/a = 4 \Rightarrow b^2 = 12$

Hence, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{12} = 1$

$\Rightarrow \quad 4x^2 - 3y^2 = 36.$

41. (c) : The path formed by football is in the shape of parabola. We know that general equation of parabola is $y^2 = 4ax$.

Since, it passes through (3, –2)

$\therefore \quad (-2)^2 = 4 \times a \times 3 \Rightarrow a = \frac{1}{3}$

$\Rightarrow$ Hence, required equation of path formed by football is $y^2 = \frac{4x}{3} \Rightarrow 3y^2 = 4x$

42. (d) : Since, $a = \frac{1}{3}$. Therefore, the equation of its directrix is $x + \frac{1}{3} = 0$.

43. (a) : The extremities of latus rectum are $(a, \pm 2a)$ $\equiv \left(\frac{1}{3}, \pm\frac{2}{3}\right)$

44. (d) : The length of latus rectum $= 4a = 4 \times \frac{1}{3} = \frac{4}{3}$

45. (d) : Point $\left(\frac{3}{4}, 1\right)$ lies on the path covered by football.

$\because \quad 3(1)^2 = 4\left(\frac{3}{4}\right) \Rightarrow 3 = 3$, which is true.

46. (b) : Here, $a = 13$, $b = 12$

$\therefore$ Vertices are $(\pm a, 0)$ *i.e*, $(\pm 13, 0)$

47. (c) : Sum of focal radii = Length of major axis = $2a$ = 26

48. (d) : Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$

49. (d) : Equation of directrices are given by $x = \pm\frac{a}{e}$

i.e., $x = \pm\frac{13}{5/13} = \pm\frac{169}{5}$

50. **(b) :** Point (0, –12) lies on the path formed by Earth around the Sun.

$\because \frac{0}{169}+\frac{144}{144}=1 \Rightarrow 1=1,$ which is true.

51. **(b) :** We have, $a = 5$ and $ae = 7$

Now, $b^2 = a^2e^2 - a^2 = 49 - 25 = 24.$

So, equation is $\frac{x^2}{25}-\frac{y^2}{24}=1.$

52. **(d) :** Length of conjungate axis = $2b = 2\times 2\sqrt{6}$

$= 4\sqrt{6}$

53. **(c) :** Eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{24}{25}}=\sqrt{\frac{49}{25}}=\frac{7}{5}$

54. **(a) :** Length of latus rectum = $\frac{2b^2}{a}=\frac{48}{5}=9.6$

55. **(c) :** Equations of directrices are given by $x=\pm\frac{a}{e}$

i.e., $x=\pm\frac{25}{7}$

56. **(d) :** We have, $9x^2 + 4y^2 - 18x - 24y + 9 = 0$

$\Rightarrow 9(x-1)^2 + 4(y-3)^2 = 36 \Rightarrow \frac{(x-1)^2}{2^2}+\frac{(y-3)^2}{3^2}=1$

Here, $b > a$

$\therefore$ Sum of focal distances of a point is $2b = 6$.

57. **(d) :** We have, $5x^2 + 9y^2 - 54y + 36 = 0$

$\Rightarrow 5x^2 + 9(y-3)^2 = 45 \Rightarrow \frac{x^2}{3^2}+\frac{(y-3)^2}{(\sqrt{5})^2}=1$

$\therefore$ Length of major axis = 2 × 3 = 6

and length of minor axis = $2\times\sqrt{5}=2\sqrt{5}$

58. **(d) :** Since, $c \geq a$, the eccentricity is never less than one. In terms of the eccentricity the foci are at a distance of ae from the centre.

59. **(d) :** A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with the axis is called the vertex of the parabola.

60. **(c) :** If the equation has a y^2 term, then the axis of symmetry is along the x-axis and if the equation has x^2 term, then the axis of symmetry is along the y-axis.

SUBJECTIVE TYPE QUESTIONS

1. As the circle is passing through the point (4, 5) and its centre is (2, 2) so its radius is $\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}.$ Therefore the required equation of circle is $(x-2)^2 + (y-2)^2 = 13$.

2. Equation of circle is $x^2 + y^2 + 2y = 0$ ...(i)

General equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, with centre $(-g, -f)$

On comparing general equation with (i), we get $2g = 0$, $2f = 2 \therefore g = 0, f = 1$

So, the centre of given circle is centre (0, –1)

3. Let the equation of the parabola with vertex at origin and axis along x-axis is $y^2 = 4ax$ ...(i)

(i) passes through (2, 8).

$\therefore$ $(8)^2 = 4a(2) \Rightarrow a = 8$

Putting value of a in (i), we get required equation of parabola $y^2 = 32x$.

4. Since, distance of the focus of a parabola from its directrix is $2a$.

$\therefore$ Given, $2a = 4 \Rightarrow a = 2$

Now, length of latus rectum = $4a = 8$.

5. Given, parabola $y^2 = 12x = 4ax$

$\Rightarrow 4a = 12$ or $a = 3$ $\therefore$ Focus = (3, 0)

6. Given, equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

Major axis is along x-axis as 25 > 9.

Here, $a^2 = 25$, $b^2 = 9$.

$\therefore$ Eccentricity (e) = $\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

7. We have $ae = 5$, $\frac{a}{e}=\frac{36}{5}$ which gives $a^2 = 36$

or $a = 6$. Therefore, $e=\frac{5}{6}.$

Now $b=a\sqrt{1-e^2}=6\sqrt{1-\frac{25}{36}}=\sqrt{11}$. Thus, the equation

of the ellipse is $\frac{x^2}{36}+\frac{y^2}{11}=1.$

8. Given, $4x^2 + y^2 = 100 \Rightarrow \frac{x^2}{25}+\frac{y^2}{100}=1$

Here, $a^2 = 100$, $b^2 = 25$

$\therefore c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

$\therefore e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}.$

9. Vertices are (±2, 0), foci are (±3, 0)

$\Rightarrow a = 2, c = 3 \quad \therefore e=\frac{c}{a}=\frac{3}{2}.$

10. Given $9y^2 - 4x^2 = 36 \Rightarrow \frac{y^2}{4}-\frac{x^2}{9}=1$

$a^2 = 4,\ b^2 = 9$

$\therefore$ Eccentricity, $e=\frac{\sqrt{a^2+b^2}}{a}=\frac{\sqrt{4+9}}{2}=\frac{\sqrt{13}}{2}.$

11. The given lines $3x - 4y + 4 = 0$ &

$2(3x - 4y - \frac{7}{2}) = 0$ are parallel to each other. And distance between given parallel lines gives the diameter of the circle.

$\therefore$ Diameter = $\left|\frac{4+\frac{7}{2}}{\sqrt{9+16}}\right|=\left(\frac{15}{2}\right)\times\frac{1}{5}=\frac{3}{2}$

$\therefore$ Radius of circle = 3/4.

12. Let equation of circle with centre (1, – 2) and radius r be $(x-1)^2+(y+2)^2=r^2$ ...(i)

and centre of given circle $x^2+y^2-4x-6y+11=0$ is (2, 3).

$\because$ (i) passes through (2, 3).

$\therefore$ $(2-1)^2+(3+2)^2=r^2$

$\Rightarrow$ $1+25=r^2 \Rightarrow r^2=26$.

Put $r^2=26$ in (i), we get

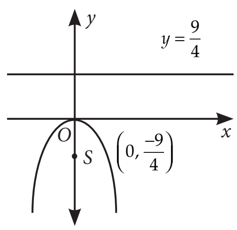
$(x-1)^2+(y+2)^2=26$

$\Rightarrow$ $x^2+1-2x+y^2+4+4y=26$

$\Rightarrow$ $x^2+y^2-2x+4y-21=0$

13. Given parabola is $x^2=-9y$ which is of the form $x^2=-4ay$

where $4a=9 \Rightarrow a=\frac{9}{4}$.



Hence, it is symmetric about y-axis.

Coordinates of focus are

$(0,-a)=\left(0,-\frac{9}{4}\right)$

Axis of the parabola is $x=0$.

Equation of directrix is $y=a$ *i.e.*, $y=\frac{9}{4}$

Length of latus rectum $=4a=4\times\left(\frac{9}{4}\right)=9$ units.

14. We have $25x^2+9y^2-150x-90y+225=0$

$\Rightarrow$ $25(x^2-6x)+9(y^2-10y)=-225$

$\Rightarrow$ $25(x^2-6x+9)+9(y^2-10y+25)=225$

$\Rightarrow$ $25(x-3)^2+9(y-5)^2=225$

$\Rightarrow$ $\frac{(x-3)^2}{9}+\frac{(y-5)^2}{25}=1$

The eccentricity is given by

$$e=\sqrt{1-\frac{a^2}{b^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

15. Let equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

The coordinates of foci are $(\pm ae, 0)$.

$\therefore$ $ae=2 \Rightarrow a\times\frac{1}{3}=2 \Rightarrow a=6$

Now, $b^2=a^2(1-e^2) \Rightarrow b^2=36\left(1-\frac{1}{9}\right)=32$

Thus, equation of required ellipse is $\frac{x^2}{36}+\frac{y^2}{32}=1$.

16. Vertices are $(\pm5, 0)$; foci $(\pm4, 0)$

Ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

We have, $a=5$, $c=4=\sqrt{a^2-b^2}$

$\Rightarrow$ $16=25-b^2 \Rightarrow b^2=9$

Substituting for a, b in (i), we get

$\frac{x^2}{25}+\frac{y^2}{9}=1$ as equation of the ellipse.

17. Let the equation of the required ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and let e be its eccentricity.

We have, $e=\frac{5}{8}$ and $2ae=10 \Rightarrow ae=5$

$\Rightarrow$ $a=8$

$\therefore$ $b^2=a^2(1-e^2) \Rightarrow b^2=64\left(1-\frac{25}{64}\right)=39$

Hence, length of the latus rectum

$=\frac{2b^2}{a}=2\times\frac{39}{8}=\frac{39}{4}$

18. Let the equation of the hyperbola be

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

Its vertices are at $(0, \pm 5)$ and foci are at $(0, \pm 8)$.

$\therefore$ $b=5$ and $be=8$.

Now, $a^2=b^2(e^2-1) \Rightarrow a^2=(be)^2-b^2=64-25=39$

Substituting the values of a and b in (i), we get

$-\frac{x^2}{39}+\frac{y^2}{25}=1$ as the equation of the hyperbola.

19. Given that the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is passing through the points (3, 0) and $(3\sqrt{2},2)$, so we get $a^2=9$ and $b^2=4$.

Again, we know that $b^2=a^2(e^2-1)$. This gives

$4=9(e^2-1) \Rightarrow e^2=\frac{13}{9} \Rightarrow e=\frac{\sqrt{13}}{3}$

20. Let hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ...(i)

Foci are $(\pm3\sqrt{5},0)$ and latus rectum is 8.

$\therefore$ $c=3\sqrt{5}$ and $\frac{2b^2}{a}=8 \Rightarrow b^2=4a$

$\therefore$ $\sqrt{a^2+b^2}=3\sqrt{5}$

$\Rightarrow$ $a^2+b^2=45$ $\Rightarrow$ $a^2+4a-45=0$

$\Rightarrow$ $(a+9)(a-5)=0$

$\therefore$ $a=5$ $\because$ $a=-9$, (not possible)

So, $b^2=4\times5=20$

Substituting for a, b in (i), we get hyperbola as

$\frac{x^2}{25}-\frac{y^2}{20}=1$.

21. Let (x_1, y_1) and (x_2, y_2) be the coordinates of points A and B respectively.

It is given that x_1, x_2 are roots of $ax^2+bx+c=0$ and y_1, y_2 are roots of $a'y^2+b'y+c'=0$.

$\therefore$ $x_1+x_2=-\frac{b}{a}$, $x_1x_2=\frac{c}{a}$,

$y_1+y_2=-\frac{b'}{a'}$ and $y_1y_2=\frac{c'}{a'}$ ...(i)

The equation of the circle with AB as diameter is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow$ $x^2+y^2-x(x_1+x_2)-y(y_1+y_2)+x_1x_2+y_1y_2=0$.

$\Rightarrow \quad x^2+y^2-x\left(-\frac{b}{a}\right)-\left(-\frac{b'}{a'}\right)y+\frac{c}{a}+\frac{c'}{a'}=0$

$\Rightarrow \quad aa'(x^2 + y^2) + a'\, bx + ab'\, y + (ca' + c'\, a) = 0$

22. Let circle be $(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Circle (i) passes through the points (2, –2) and (3, 4).

$\therefore \quad (2 - h)^2 + (-2 - k)^2 = r^2$ ...(ii)

and $(3 - h)^2 + (4 - k)^2 = r^2$ ...(iii)

Also centre (h, k) lies on $x + y = 1$

$\therefore \quad h + k = 1$ ...(iv)

From (ii) and (iii), we get

$(2 - h)^2 + (-2 - k)^2 = (3 - h)^2 + (4 - k)^2$

$\Rightarrow \quad 4 - 4h + h^2 + 4 + 4k + k^2 = 9 - 6h + h^2 + 16 - 8k + k^2$

$\Rightarrow \quad 2h + 12k = 17$ ...(v)

Solving (iv) and (v), we get, $h=-\frac{1}{2}, k=\frac{3}{2}$

Substituting for h, k in (iii), we get

$$\left(3+\frac{1}{2}\right)^2+\left(4-\frac{3}{2}\right)^2=r^2$$

$$\Rightarrow \quad \frac{49}{4}+\frac{25}{4}=r^2 \Rightarrow \frac{74}{4}=r^2 \Rightarrow r=\frac{\sqrt{74}}{2}$$

Substituting for h, k, r in (i), we get

$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{74}{4}$$

$$\Rightarrow \quad x^2+x+\frac{1}{4}+y^2-3y+\frac{9}{4}=\frac{74}{4}$$

$\Rightarrow \quad x^2 + y^2 + x - 3y - 16 = 0$ is the required equation.

23. Let equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\because \quad$ (4, 1) and (6, 5) lie on (i).

$\therefore \quad (4)^2 + (1)^2 + 2(4)g + 2(1)f + c = 0$

$\Rightarrow \quad 8g + 2f + c = -17$ ...(ii)

and $(6)^2 + (5)^2 + 2(6)g + 2(5)f + c = 0$

$\Rightarrow \quad 12g + 10f + c = -61$ ...(iii)

(iii) – (ii) gives $4g + 8f = -44$ ...(iv)

Also, centre $(-g, -f)$ lies on $4x + y = 16$.

$\therefore \quad -4g - f = 16$ ...(v)

Solving (iv) and (v), $g = -3, f = -4$

Putting values of g and f in (ii) we get

$8(-3) + 2(-4) + c = -17 \Rightarrow c = 15$

$\therefore \quad$ Required equation of circle is

$x^2 + y^2 - 6x - 8y + 15 = 0$

24. Let equation of required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\because \quad$ (3, 7) and (5, 5) lie on (i).

$\therefore \quad (3)^2 + (7)^2 + 2(3)g + 2(7)f + c = 0$

$\Rightarrow \quad 6g + 14f + c = -58$ ...(ii)

and $(5)^2 + (5)^2 + 2(5)g + 2(5)f + c = 0$

$\Rightarrow \quad 10g + 10f + c = -50$ ...(iii)

(iii) – (ii) gives $4g - 4f = 8 \Rightarrow g - f = 2$ ...(iv)

Also, centre $(-g, -f)$ lies on $x - 4y = 1$

$\therefore \quad -g - 4(-f) = 1$

or $\quad -g + 4f = 1$ ...(v)

Solving (iv) and (v), $f = 1, g = 3$

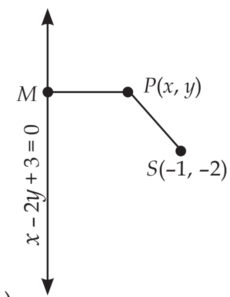
Putting these values in (iii), we get

$30 + 10 + c = -50 \Rightarrow c = -90$

$\because \quad$ Required equation of circle is

$x^2 + y^2 + 6x + 2y - 90 = 0.$

25. Let $P(x, y)$ be any point on the parabola whose focus is $S(-1, -2)$ and the directrix $x - 2y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 2y + 3 = 0$.



Then, by definition

$SP = PM \quad \Rightarrow \quad SP^2 = PM^2$

$$\Rightarrow \quad (x+1)^2+(y+2)^2=\left(\frac{x-2y+3}{\sqrt{1+4}}\right)^2$$

$\Rightarrow \quad 5[(x + 1)^2 + (y + 2)^2]$

$= (x - 2y + 3)^2$

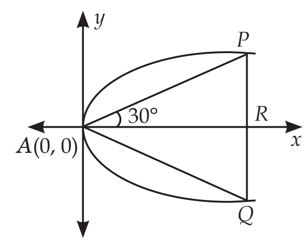
$\Rightarrow \quad 5(x^2 + y^2 + 2x + 4y + 5)$

$= (x^2 + 4y^2 + 9 - 4xy + 6x - 12y)$

$\Rightarrow \quad 4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$ is the equation of the required parabola.

26. Let ΔAPQ be an equilateral triangle with side of length l.

Since each angle of an equilateral triangle is 60°. Then $\angle PAR = 30°$



$$\cos 30° = \frac{AR}{l}$$

$$\Rightarrow \quad AR = l\frac{\sqrt{3}}{2}$$

and $PR = \frac{l}{2}$

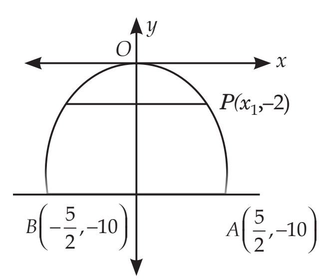
Thus, $\left(\frac{l\sqrt{3}}{2}, \frac{l}{2}\right)$ are the coordinates of point P.

Since point P also lies on the parabola $y^2 = 4ax$

$$\therefore \quad \left(\frac{l}{2}\right)^2 = 4a\times\left(\frac{l\sqrt{3}}{2}\right) \Rightarrow l = 8a\sqrt{3}$$

Hence, length of the side of the triangle is $8a\sqrt{3}$.

27.



As the axis of parabola is vertical, its equation is $x^2 = 4ay$

$\because \quad$ Point $\left(\frac{5}{2}, -10\right)$ lies on it $\therefore \quad \frac{25}{4} = 4a(-10)$

$$\Rightarrow \; 4a = \frac{-5}{8} \;\Rightarrow\; x^2 = \frac{-5}{8}y$$

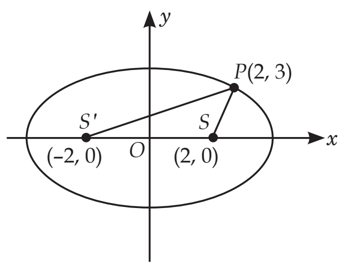
Let point $P(x_1, -2)$ lies on parabola.

$\therefore \quad (x_1)^2 = -\frac{5}{8} \times (-2) \Rightarrow x_1^2 = \frac{5}{4} \Rightarrow x_1 = \pm\frac{\sqrt{5}}{2}$

$\therefore$ Width of parabola 2 m from vertex is

$2 \times \frac{\sqrt{5}}{2} = \sqrt{5}$ m $= 2.23$ m

28. Let equation of ellipse with centre at origin and major axis along x-axis be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)



Now, $S'P + SP = 2a$

$\Rightarrow \sqrt{\{2-(-2)\}^2 + (3-0)^2} + \sqrt{(2-2)^2 + (3-0)^2} = 2a$

$\Rightarrow \quad \sqrt{16+9} + \sqrt{9} = 2a \Rightarrow 2a = 5 + 3 = 8$

$\Rightarrow \quad a = 4$

Also, foci $= (\pm 2, 0) = \left(\pm\sqrt{a^2 - b^2}, 0\right)$

$\Rightarrow \quad a^2 - b^2 = 4 \Rightarrow 16 - b^2 = 4 \Rightarrow b^2 = 12$

Putting values of a^2 and b^2 in (i), we get

$\frac{x^2}{16} + \frac{y^2}{12} = 1$, which is the required equation of ellipse.

29. Since foci of ellipse is $(0, \pm 6)$

$\therefore$ Major axis lies along y-axis.

Let equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ b > a$ ...(i)

Also, length of minor axis $= 2a = 16 \Rightarrow a = 8$

$\therefore \quad b^2 - a^2 = 36 \Rightarrow b^2 - 64 = 36 \Rightarrow b = 10$

Putting the values of a and b in (i), we get

$\frac{x^2}{64} + \frac{y^2}{100} = 1$, which is the required equation of ellipse.

Points where ellipse cuts y-axis = vertex $= (0, \pm b) = (0, \pm 10)$.

Latus rectum $= \frac{2a^2}{b} = \frac{2(64)}{10} = 12.8$ units

30. Let equation of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b \quad \text{...(i)}$$

Since, (4, 3) and (1, 4) lies on it.

$\therefore \quad \frac{16}{a^2} + \frac{9}{b^2} = 1$ ...(ii) and $\frac{1}{a^2} + \frac{16}{b^2} = 1$ ...(iii)

From (ii) and (iii) we get, $\frac{16}{a^2} + \frac{9}{b^2} = \frac{1}{a^2} + \frac{16}{b^2}$

$\Rightarrow \quad \frac{15}{a^2} = \frac{7}{b^2} \Rightarrow a^2 = \frac{15}{7}b^2$ ...(iv)

Using (iv) in (ii), we get

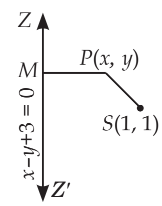
$\frac{16}{15b^2} \times 7 + \frac{9}{b^2} = 1 \Rightarrow \frac{247}{15b^2} = 1 \Rightarrow b^2 = \frac{247}{15}$ ...(v)

Using (v) in (iv), we get $a^2 = \frac{247}{7}$

Putting required values in (i), we get

$\frac{x^2}{(247/7)} + \frac{y^2}{(247/15)} = 1 \Rightarrow 7x^2 + 15y^2 = 247$

31. Let $P(x, y)$ be a point on the ellipse. Then, by definition $SP = ePM$



$\therefore \quad SP = \frac{1}{2}PM$

$\Rightarrow \quad \sqrt{(x-1)^2 + (y-1)^2} = \frac{1}{2}\left|\frac{x-y+3}{\sqrt{1^2+(-1)^2}}\right|$

$\Rightarrow \quad 8[(x-1)^2 + (y-1)^2] = (x - y + 3)^2$

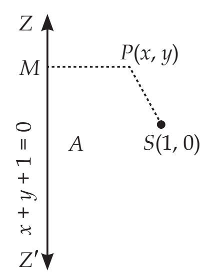
$\Rightarrow \quad 7x^2 + 7y^2 + 2xy - 22x - 10y + 7 = 0$ is the required equation of the ellipse.

The major axis is a line perpendicular to the directrix and passing through the focus.

Therefore, the equation of the major axis is

$y - 1 = -1(x - 1) \Rightarrow x + y - 2 = 0.$

32. Let $S(1, 0)$ be the focus and ZZ' be the directrix. Let $P(x, y)$ be any point on the ellipse and PM be perpendicular from P on the directrix. Then, by definition



$SP = e \cdot PM$, where $e = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad SP^2 = e^2 PM^2$

$\Rightarrow \quad (x-1)^2 + (y-0)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \left|\frac{x+y+1}{\sqrt{1+1}}\right|^2$

$\Rightarrow \quad 4[x-1)^2 + y^2] = (x + y + 1)^2$

$\Rightarrow \quad 4x^2 + 4y^2 - 8x + 4 = x^2 + y^2 + 1 + 2xy + 2x + 2y$

$\Rightarrow \quad 3x^2 + 3y^2 - 2xy - 10x - 2y + 3 = 0$

This is the equation of the required ellipse.

33. Let equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Given, $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

But, $b^2 = a^2(e^2 - 1)$

$\therefore \quad a^2(e^2 - 1) = 4a \Rightarrow a(e^2 - 1) = 4$

$\Rightarrow \quad a\left(\frac{9}{5} - 1\right) = 4 \Rightarrow a = 5 \quad \therefore \quad b^2 = 4 \times 5 = 20$

Thus, equation of required hyperbola is

$\frac{x^2}{25} - \frac{y^2}{20} = 1.$

34. The equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1.$

$\therefore \quad a^2 = 25$ and $b^2 = 9$

Eccentricity of ellipse $=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

So, the coordinates of foci are $(\pm 4, 0)$

Since the coordinates of foci of hyperbola coincide with the foci of the ellipse. So the coordinates of foci of hyperbola are $(\pm 4, 0)$.

Let equation of required hyperbola is $\frac{x^2}{a'^2}-\frac{y^2}{b'^2}=1$

Let e' be the eccentricity of required hyperbola.

then $a'e' = 4 \Rightarrow 2a' = 4 \Rightarrow a' = 2$

$\therefore \quad b'^2 = a'^2(e'^2 - 1) \Rightarrow b'^2 = 4(4 - 1) = 12$

Thus, equation of required hyperbola is

$\frac{x^2}{4}-\frac{y^2}{12}=1.$

35. Let $2a$ and $2b$ be the transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes.

Then, the equation of the hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{...(i)}$$

We have, $2b = 5$ and $2ae = 13$.

Now, $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = a^2e^2 - a^2$

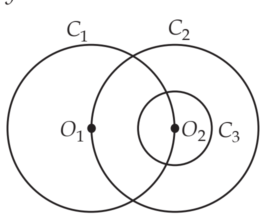
$\Rightarrow \quad \frac{25}{4}=\frac{169}{4}-a^2 \Rightarrow a^2=\frac{144}{4} \Rightarrow a=6.$

Substituting the values of a and b in (i), the equation of the hyperbola is

$\frac{x^2}{36}-\frac{y^2}{25/4}=1 \Rightarrow 25x^2-144y^2=900.$

36. We have to find the equation of circle (C_2) which passes through the centre of circle (C_1) and is concentric with circle (C_3).

$x^2 + y^2 + 8x + 10y - 7 = 0 \qquad \text{...(i)}$



$\therefore \quad$ Centre of C_1 is $O_1 = (-4, -5)$

Now, equation of circle (C_2) which is concentric with given circle (C_3) $2x^2 + 2y^2 - 8x - 12y - 9 = 0$ is

$$2x^2 + 2y^2 - 8x - 12y + k = 0 \qquad \text{...(ii)}$$

Since, circle (C_2) passes through O_1 $(-4, -5)$.

$\therefore \quad 2(-4)^2 + 2(-5)^2 - 8(-4) - 12(-5) + k = 0$

$\Rightarrow \quad 32 + 50 + 32 + 60 + k = 0 \Rightarrow k = -174$

On putting the value of k in (ii), we get

$$2x^2 + 2y^2 - 8x - 12y - 174 = 0$$

$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 87 = 0$ which is required equation of circle (C_2).

37. The given equation is $y^2 = 4x + 4y$

$\Rightarrow \quad y^2 - 4y = 4x$

$\Rightarrow \quad y^2 - 4y + 4 = 4x + 4$

$\Rightarrow \quad (y - 2)^2 = 4(x + 1) \qquad \text{...(i)}$

Shifting the origin to the point $(-1, 2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have,

$$x = X + (-1),\ y = Y + 2 \qquad \text{...(ii)}$$

Using these relations, (i) reduces to

$$Y^2 = 4X$$

This is of the form $Y^2 = 4aX$.

On comparing we get $4a = 4 \Rightarrow a = 1$.

Vertex: The coordinates of the vertex w.r.t. new axes are $(X = 0, Y = 0)$.

The coordinates of the vertex w.r.t. old axes are $(-1, 2)$ [Putting $X = 0$, $Y = 0$ in (ii)]

Focus: The coordinates of the focus w.r.t. new axes are $(1, 0)$.

So, coordinates of the focus w.r.t. old axes are $(0, 2)$ [Putting $X = 1$, $Y = 0$ in (ii)]

Directrix: Equation of the directrix of the parabola w.r.t. new axes is $X = -1$

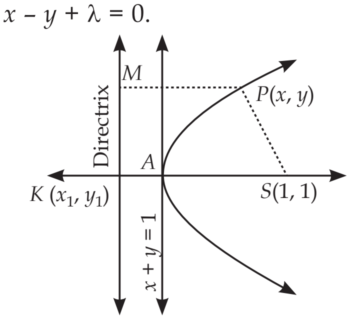
So, equation of the directrix of the parabola w.r.t. old axes is $x = -2$.

Axis : Equation of axis of the parabola w.r.t. new axes is $Y = 0$

So, equation of axis w.r.t. old axes is $y = 2$ [Putting $Y = 0$ in (ii)]

Latus rectum: The length of the latus rectum = 4.

38. Let S be the focus and A be the vertex of the parabola. Let K be the point of intersection of the axis and directrix. Since axis is a line passing through $S(1, 1)$ and perpendicular to $x + y = 1$. So, let the equation of the axis be $x - y + \lambda = 0$.



This will pass through $S(1, 1)$, if

$1 - 1 + \lambda = 0 \Rightarrow \lambda = 0$

So the equation of the axis is

$x - y = 0$

The vertex A is the point of intersection of $x - y = 0$ and $x + y = 1$. Solving these two equations, we get $x = 1/2$ and $y = 1/2$.

Let (x_1, y_1) be the coordinates of K. As A is the mid-point of SK.

$\therefore \quad \frac{x_1+1}{2}=\frac{1}{2}, \frac{y_1+1}{2}=\frac{1}{2} \Rightarrow x_1=0, y_1=0$

So, the coordinates of K are $(0,0)$. Since directrix is a line

passing through K (0,0) and parallel to $x + y = 1$. Therefore, equation of the directrix is
$y - 0 = -1 (x - 0)$ or, $x + y = 0$
Let $P(x, y)$ be any point on the parabola. Then,

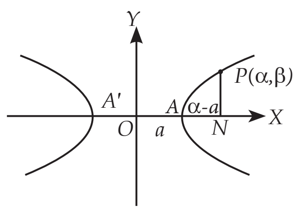
Distance of P from the focus S
= Distance of P from the directrix $x + y = 0$

$$\Rightarrow \sqrt{(x-1)^2+(y-1)^2} = \left|\frac{x+y}{\sqrt{1^2+1^2}}\right|$$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2xy$
$\Rightarrow \quad x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$, which is the required equation of the parabola.

39. Let P (α, β) be any point on hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{...(i)}$$



Then, $\dfrac{\alpha^2}{a^2}-\dfrac{\beta^2}{b^2}=1$

$$\Rightarrow \frac{\alpha^2}{a^2}-1=\frac{\beta^2}{b^2}$$

$$\Rightarrow \frac{\alpha^2-a^2}{a^2}=\frac{\beta^2}{b^2}$$

$$\Rightarrow \frac{(\alpha-a)(\alpha+a)}{a^2}=\frac{\beta^2}{b^2}$$

$$\Rightarrow \frac{(AN)(A'N)}{a^2}=\frac{(PN)^2}{b^2} \quad \Rightarrow \frac{(PN)^2}{(AN)(A'N)}=\frac{b^2}{a^2}$$

40. Let the equation of the hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \textbf{...(i)}$$

Then, the equation of the hyperbola conjugate to (i) is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \qquad \text{...(ii)}$$

We have, e = Eccentricity of (i)

$$e=\sqrt{1+\left(\frac{\text{conjugate axis}}{\text{transverse axis}}\right)^2}$$

$$\Rightarrow \quad e=\sqrt{1+\left(\frac{2b}{2a}\right)^2} \Rightarrow e^2=1+\frac{b^2}{a^2} \Rightarrow e^2=\frac{a^2+b^2}{a^2} \qquad \text{...(iii)}$$

and, e' = Eccentricity of (ii)

$$=\sqrt{1+\left(\frac{\text{conjugate axis}}{\text{transverse axis}}\right)^2}$$

$$\Rightarrow \quad e'=\sqrt{1+\left(\frac{2a}{2b}\right)^2} \Rightarrow e'^2=1+\frac{a^2}{b^2}$$

$$\Rightarrow \quad e'^2=\frac{a^2+b^2}{b^2} \qquad \text{...(iv)}$$

From (iii) and (iv), we have

$$\frac{1}{e^2}+\frac{1}{e'^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}$$

$$\Rightarrow \quad \frac{1}{e^2}+\frac{1}{e'^2}=1$$